

The Universal Law of the Excluded Middle  
or  
Weapons of Mass Instruction  
Cary Academy Debate: 8/12/5

**Claim Types**

SYNTHETIC	ANALYTIC
A POSTERIORI	A PRIORI

ANALYTIC: An analytic claim is a claim the truth or falsity of which is verifiable by means only of language (definitions).

SYNTHETIC: A synthetic claim is a claim that is **NOT** ANALYTIC.

A POSTERIORI: An a posteriori claim is a claim the truth or falsity of which is verifiable by means only of empirical (sensory) evidence.

A PRIORI: An a priori claim is a claim that is **NOT** A POSTERIORI.

**Refutation Types**

NON: A NON refutation is a refutation that asserts the strictly ANALYTIC opposite of the claim to be refuted. For example, if the given claim is “This chair is red,” then the NON refutation is “This chair is **not** red.” More rigorously, the NON refutation would be “It is **not** the case that this chair is red.” For any given claim, there is one and only one possible NON refutation.

POP: A pop refutation is a refutation that asserts some sort of SYNTHETIC opposite of the claim to be refuted. For example, if the given claim is “This chair is red,” then on example of a POP refutation might be “This chair is green.” For any given claim, there may be a wide array of possible POP refutations.

## TRUTH TABLES

<u>P</u>	<u>Q</u>	<u>~P</u>	<u>~Q</u>	<u>P&amp;Q</u>
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F
<u>~(P&amp;Q)</u>	<u>PvQ</u>	<u>~(PvQ)</u>	<u>P-&gt;Q</u>	<u>Q-&gt;P</u>
F	T	F	T	T
F	F	T	F	F
T	T	F	T	T
T	F	T	F	F
<u>~P-&gt;~Q</u>	<u>~Q-&gt;~P</u>	<u>P&lt;-&gt;Q</u>	<u>P&amp;~P</u>	<u>Pv~P</u>
T	T	T	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

## PROOFS

### 1. $P \rightarrow Q, P \vdash Q$                      [ $\rightarrow$ : Modus Ponens]

1.	$P \rightarrow Q$	A	1
2.	P	A	2
3.	Q	$\rightarrow$ o 1,2	1,2
			QED

\* *NOTE: This is*      *NEITHER*       $P \rightarrow Q, Q$        $\vdash P$   
                                  *NOR*               $P \rightarrow Q, \sim P$        $\vdash \sim Q$   
                                  *NOR*               $P \rightarrow Q$                $\vdash Q$ . See Truth Table for  $\rightarrow$ .

### 2. $P \& Q \vdash P$                                      [ $\&$ o]

1.	$P \& Q$	A	1
2.	P	$\&$ o 1	1
			QED

### 3. $P, Q \vdash P \& Q$                                      [ $\&$ i]

1.	P	A	1
2.	Q	A	2
3.	$P \& Q$	$\&$ i 1,2	1,2
			QED

### 4. $P \rightarrow Q, P \& R \vdash Q$                                      [ $\rightarrow$ o, $\&$ o]

1.	$P \rightarrow Q$	A	1
2.	$P \& R$	A	2
3.	P	$\&$ o 2	2
4.	Q	$\rightarrow$ o 1,3	1,2
			QED

### 5. $P \vdash P \vee Q$    [ $\vee$ i]

1.	P	A	1
2.	$P \vee Q$	$\vee$ i 1	1
			QED

**6.  $P \rightarrow Q, P \& R \mid - Q$**

**[ $\sim o$ ]**

1.	$P \rightarrow Q$	A	1
2.	$P \& R$	A	2
3.	$\sim Q$	PA	3
4.	P	PPA	4
5.	Q	$\rightarrow o$ 1,4	1,4
6.	$Q \& \sim Q$	$\& i$ 3,5	1,3,4
7.	$\sim P$	$\sim i$ 4-6	1,3
8.	P	$\& o$ 2	2
9.	$P \& \sim P$	$\& i$ 7,8	1,2,3
10.	Q	$\sim o$ 3-9	1,2
			QED

**7.  $A \vee \sim A$**

**[ $\sim o$ : ULEM form #2]**

1.	$\sim(A \vee \sim A)$	PA	1
2.	A	PPA	2
3.	$A \vee \sim A$	$\vee i$ 2	2
4.	$(A \vee \sim A) \& \sim(A \vee \sim A)$	$\& i$ 1,3	1,2
5.	$\sim A$	$\sim i$ 2-4	1
6.	$\sim A$	PPA	6
7.	$A \vee \sim A$	$\vee i$ 6	6
8.	$(A \vee \sim A) \& \sim(A \vee \sim A)$	$\& i$ 1,7	1,6
9.	A	$\sim o$ 6-8	1
10.	$A \& \sim A$	$\& i$ 5,9	1
11.	$A \vee \sim A$	$\sim o$ 1-10	---
			QED

**8.  $P \rightarrow R, Q \rightarrow R, P \vee Q \mid - R$**

**[ $\vee o$ ]**

1.	$P \rightarrow R$	A	1
2.	$Q \rightarrow R$	A	2
3.	$P \vee Q$	A	3
4.	R	$\vee o$ 1,2,3	1,2,3
			QED

**9.  $A \rightarrow P, \sim A \rightarrow P \mid - P$**

**[vo: ULEM form #2]**

1.	$A \rightarrow P$	A	1
2.	$\sim A \rightarrow P$	A	2
3.	$\sim(A \vee \sim A)$	PA	3
4.	A	PPA	4
5.	$A \vee \sim A$	vi 4	4
6.	$(A \vee \sim A) \& \sim(A \vee \sim A)$	&i 3,5	3,4
7.	$\sim A$	$\sim i$ 4-6	3
8.	$\sim A$	PPA	8
9.	$A \vee \sim A$	vi 8	8
10.	$(A \vee \sim A) \& \sim(A \vee \sim A)$	&i 3,9	3,8
11.	A	$\sim o$ 8-10	3
12.	$A \& \sim A$	&i 7,11	3
13.	$A \vee \sim A$	$\sim o$ 3-12	---
14.	P	vo 1,2,13	1,2
			QED

**10.  $P \rightarrow (P \vee Q)$**

**[ $\rightarrow i$ ]**

1.	P	PA	1
2.	$P \vee Q$	vi 1	1
3.	$P \rightarrow (P \vee Q)$	$\rightarrow i$ 1-2	---
			QED

**11.  $P \rightarrow Q, Q \rightarrow P \mid - P \leftrightarrow Q$**

**[ $\leftrightarrow i$ ]**

1.	$P \rightarrow Q$	A	1
2.	$Q \rightarrow P$	A	2
3.	$(P \rightarrow Q) \& (Q \rightarrow P)$	&i 1,2	1,2
4.	$P \leftrightarrow Q$	$\leftrightarrow i$ 3	1,2
			QED

**12.  $P \leftrightarrow Q \mid - P \rightarrow Q$**

**[ $\leftrightarrow o$ ]**

1.	$P \leftrightarrow Q$	A	1
2.	$P \rightarrow Q$	$\leftrightarrow o$ 1	1
			QED

**13.  $P \rightarrow Q \mid \sim PvQ$**

**[ $\rightarrow \mid \sim v$ ]**

1.	$P \rightarrow Q$	A	1
2.	$\sim(\sim PvQ)$	PA	2
3.	P	PPA	3
4.	Q	$\rightarrow o$ 1,3	1,3
5.	$\sim PvQ$	vi 4	1,3
6.	$(\sim PvQ) \& \sim(\sim PvQ)$	$\& i$ 2,5	1,2,3
7.	$\sim P$	$\sim i$ 3-6	1,2
8.	$\sim P$	PPA	8
9.	$\sim PvQ$	vi 8	8
10.	$(\sim PvQ) \& \sim(\sim PvQ)$	$\& i$ 2,9	2,8
11.	P	$\sim o$ 8-10	2
12.	$P \& \sim P$	$\& i$ 7,11	1,2
13.	$(\sim PvQ)$	$\sim o$ 2-12	1
			QED

**14.  $(\sim PvQ) \rightarrow (P \rightarrow Q)$**

**[ $v \mid \rightarrow$ ]**

1.	$\sim PvQ$	PA	1
2.	P	PPA	2
3.	$\sim P$	PPPA	3
4.	$\sim Q$	PPPPA	4
5.	$P \& \sim P$	$\& i$ 2,3	2,3
6.	Q	$\sim o$ 4-5	2,3
7.	$\sim P \rightarrow Q$	$\rightarrow i$ 3-6	2
8.	Q	PPPA	8
9.	$Q \& P$	$\& i$ 2,8	2,8
10.	Q	$\& o$ 9	2,8
11.	$Q \rightarrow Q$	$\rightarrow i$ 8-10	2
12.	Q	vo 1,7,11	1,2
13.	$P \rightarrow Q$	$\rightarrow i$ 2-12	1
14.	$(\sim PvQ) \rightarrow (P \rightarrow Q)$	$\rightarrow i$ 1-13	---
			QED

**15.  $\sim(P \& Q) \vdash \sim P \vee \sim Q$**

**[ $\&$   $\vdash$  v: DeMorgan's Law form #1]**

1.	$\sim(P \& Q)$	A	1
2.	$\sim(\sim P \vee \sim Q)$	PA	2
3.	$\sim P$	PPA	3
4.	$\sim P \vee \sim Q$	vi 3	3
5.	$(\sim P \vee \sim Q) \& \sim(\sim P \vee \sim Q)$	$\&i$ 2,4	2,3
6.	P	$\sim o$ 3-5	2
7.	$\sim Q$	PPA	7
8.	$\sim P \vee \sim Q$	vi 7	7
9.	$(\sim P \vee \sim Q) \& \sim(\sim P \vee \sim Q)$	$\&i$ 2,8	2,7
10.	Q	$\sim o$ 7-9	2
11.	$P \& Q$	$\&i$ 6,10	2
12.	$(P \& Q) \& \sim(P \& Q)$	$\&i$ 1,11	1,2
13.	$\sim P \vee \sim Q$	$\sim o$ 2-12	1
			QED

**16.  $(\sim P \vee \sim Q) \rightarrow \sim(P \& Q)$**

**[ $\vee$   $\vdash$   $\&$ : Converse, DeMorgan's Law form #1]**

1.	$\sim P \vee \sim Q$	PA	1
2.	$P \& Q$	PPA	2
3.	P	$\&o$ 2	2
4.	$\sim P$	PPPA	4
5.	Q	PPPPA	5
6.	$P \& \sim P$	$\&i$ 3,4	2,4
7.	$\sim Q$	$\sim i$ 5-6	2,4
8.	$\sim P \rightarrow \sim Q$	$\rightarrow i$ 4-7	2
9.	$\sim Q$	PPPA	9
10.	$\sim Q \& (\sim P \vee \sim Q)$	$\&i$ 1,9	1,9
11.	$\sim Q$	$\&o$ 10	1,9
12.	$\sim Q \rightarrow \sim Q$	$\rightarrow i$ 9-11	1
13.	$\sim Q$	$\vee o$ 1,8,12	1,2
14.	Q	$\&o$ 2	2
15.	$Q \& \sim Q$	$\&i$ 13,14	1,2
16.	$\sim(P \& Q)$	$\sim i$ 2-15	1
17.	$(\sim P \vee \sim Q) \rightarrow \sim(P \& Q)$	$\rightarrow i$ 1-16	---
			QED

**17.  $(P \rightarrow Q) \rightarrow \sim(P \& \sim Q)$**

[ $\rightarrow$  |  $\&$ ]

1.	$P \rightarrow Q$	PA	1
2.	$P \& \sim Q$	PPA	2
3.	P	$\& o 2$	2
4.	$\sim Q$	$\& o 2$	2
5.	Q	$\rightarrow o 1,3$	1,2
6.	$Q \& \sim Q$	$\& i 4,5$	1,2
7.	$\sim(P \& \sim Q)$	$\sim i 2-6$	1
8.	$(P \rightarrow Q) \rightarrow \sim(P \& \sim Q)$	$\rightarrow i 1-7$	---
			QED

**18.  $\sim(P \& \sim Q) \rightarrow (P \rightarrow Q)$**

[ $\&$  |  $\rightarrow$ ]

1.	$\sim(P \& \sim Q)$	PA	1
2.	P	PPA	2
3.	$\sim Q$	PPPA	3
4.	$P \& \sim Q$	$\& i 2,3$	2,3
5.	$(P \& \sim Q) \& \sim(P \& \sim Q)$	$\& i 1,4$	1,2,3
6.	Q	$\sim o 3-5$	1,2
7.	$P \rightarrow Q$	$\rightarrow i 2-6$	1
8.	$[\sim(P \& \sim Q)] \rightarrow (P \rightarrow Q)$	$\rightarrow i 1-7$	---
			QED

**19.  $(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$**

[ $\rightarrow$  |  $\leftarrow$ : Contrapositive]

1.	$P \rightarrow Q$	PA	1
2.	$\sim Q$	PPA	2
3.	P	PPPA	3
4.	Q	$\rightarrow o 1,3$	1,3
5.	$Q \& \sim Q$	$\& i 2,4$	1,2,3
6.	$\sim P$	$\sim i 3-5$	1,2
7.	$\sim Q \rightarrow \sim P$	$\rightarrow i 2-6$	1
8.	$(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$	$\rightarrow i 1-7$	---
			QED